Triaxial Deformation and Asynchronous Rotation of Rocky Planets in the Habitable Zone of Low-Mass Stars

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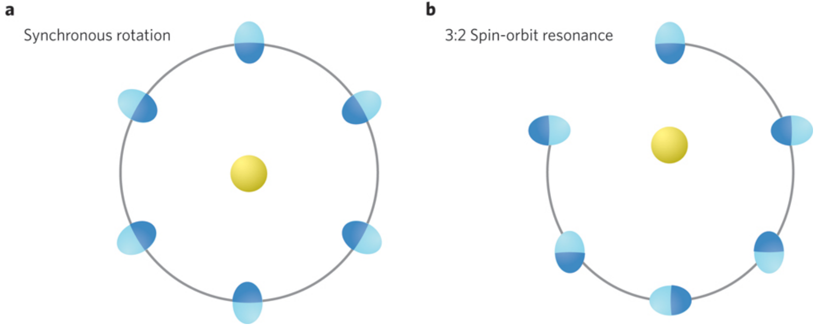
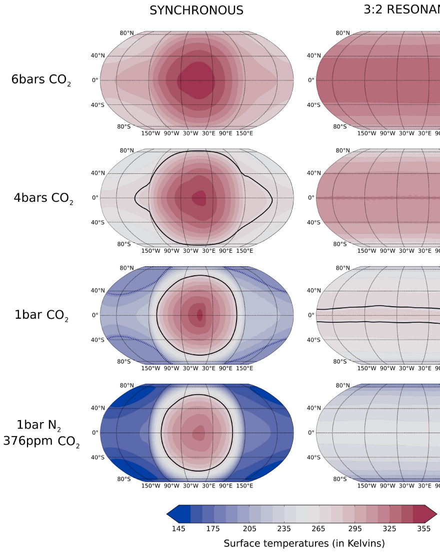
Spin-Orbit Resonance

Rocky planets close to their host stars may be locked into spin-orbit resonances.



Implications for Habitability



Turbet et al. (2016), looking at the habitability of Proxima Centauri b.

Spin-Orbit Resonant Capture

Two torques acting on planet's spin angular momentum near 3:2 spin-orbit resonance:

$$|\mathbf{T}_{\rm tri}| \simeq \frac{21}{4}\epsilon I e \left(\frac{GM_\star}{a^3}\right), \qquad |\mathbf{T}_{\rm tide}| \simeq \frac{3GM_\star^2 R^5}{a^6}\frac{k_2}{Q},$$

where $\epsilon = (I_{yy} - I_{xx})/I_{zz}$, with $I_{zz} \geq I_{yy} \geq I_{xx}$. For spin-orbit resonant capture, require $|\mathbf{T}_{\rm tri}| > |\mathbf{T}_{\rm tide}|$:

$$\Rightarrow \epsilon > 1.0 \times 10^{-6} \left(\frac{M_\star}{0.3\,M_\odot}\right)\left(\frac{a}{0.1\,\mathrm{AU}}\right)^{-3} \times \left(\frac{\rho}{6\,\mathrm{g/cm^3}}\right)^{-1}\left(\frac{k_2/Q}{10^{-3}}\right)\left(\frac{e}{0.01}\right)^{-1}. \quad (1)$$

What is Maximal ellipticity?

Lower bound on $\epsilon = (I_{yy} - I_{xx})/I_{zz}$ set by tides. What is upper bound on ϵ, set by Geophysics? May super-earths achieve values of ϵ capable of overcoming the lower limit set by tides?

Order of Magnitude estimate of $\epsilon_{\max}$

The two stresses on the planet's core are

$$|\mathbf{T}_{\rm grav}| \sim \rho g R \epsilon, \qquad |\mathbf{T}_{\rm elast}| \sim \mu u.$$

$\mu =$ shear modulus, $u =$ strain. Requiring $|\mathbf{T}_{\rm elast}| \approx |\mathbf{T}_{\rm grav}|$ gives

$$u \sim \frac{\rho g R \epsilon}{\mu}.$$

Von-Mises criterion: Crust yields when $u \geq u_{\rm crit} \sim 10^{-5}$. In terms of ellipticity:

$$\epsilon \geq \epsilon_{\max} \sim \left(\frac{\mu}{\rho g R}\right) u_{\rm crit}.$$

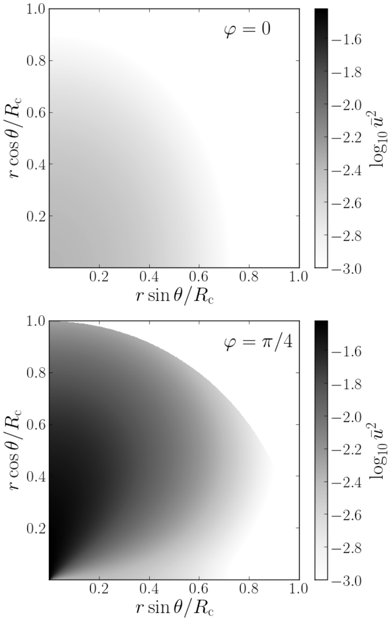
Realistic model adds additional $\sim 1/8$ geometric factor.

Detailed Analytic Model

Assumptions:

- Constant density core $\rho = \rho_{\rm c}$
- Homogeneous $\mu =$ constant
- Incompressible
- Fluid envelope with isothermal or constant density equation of state

Bare Core



$$\bar{u} \sim \left(\frac{\mu}{\rho g R \epsilon}\right) u.$$

$$\bar{u}_{\mathrm{peak}} = \max(\bar{u}) = 0.195.$$

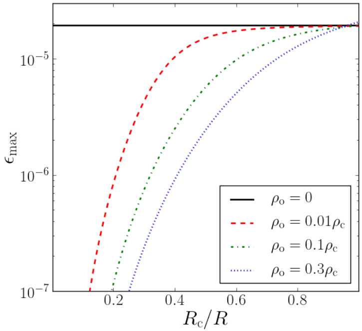
$$\Rightarrow u_{\mathrm{peak}} \simeq \frac{1}{7.9}\frac{\rho g R}{\mu}\epsilon.$$

Setting $u_{\mathrm{peak}} = u_{\mathrm{crit}}$:

$$\epsilon_{\mathrm{max}} = 1.9 \times 10^{-5}\left(\frac{\mu}{10^{12}\,\mathrm{dyn/cm^2}}\right) \times \left(\frac{\rho}{6\,\mathrm{g/cm^3}}\right)^{-2}\left(\frac{R}{R_\oplus}\right)^{-2}\left(\frac{u_{\mathrm{crit}}}{10^{-5}}\right).$$

Fluid Envelope

$\epsilon_{\max}$ for constant density ocean.



Thin Isothermal atmosphere, equation of state $\rho = p/c_s^2$, $c_s^2 \ll g_c R_c$.

$$\epsilon_{\max} = \left(1 - \frac{\rho_a}{\rho_c}\right)^{-1} \epsilon_{\text{max, bare core}}.$$

Constant density ocean $\rho = \rho_o$:

$$\epsilon_{\max} = H\left(\frac{R_c}{R}, \frac{\rho_o}{\rho_c}\right) \epsilon_{\text{max, bare core}}.$$

Comparison to Solar System Values

Body	Observed ϵ	$\epsilon_{\rm max}$
Mercury	1.3×10^{-4}	1.6×10^{-4}
Venus	5.4×10^{-6}	2.9×10^{-5}
Earth	1.5×10^{-5}	2.3×10^{-5}
Mars	5.5×10^{-4}	1.6×10^{-4}
Moon	2.3×10^{-4}	8.7×10^{-4}

Observed ϵ computed through (Yoder 1995)

$$\epsilon \approx \frac{I_{yy} - I_{xx}}{2MR^2/5} = 10C_{22},$$

where C_{22} is gravitational coefficient.

Conclusions

- Maximal Triaxiality of rocky planet given by

$$\epsilon_{\rm max} = 1.9 \times 10^{-5} \left(\frac{\mu}{10^{12}\,\mathsf{dyn/cm^2}} \right) \left(\frac{\rho}{6\,\mathsf{g/cm^3}} \right)^{-2} \times \left(\frac{R}{R_\oplus} \right)^{-2} \left(\frac{u_{\rm crit}}{10^{-5}} \right).$$

- Fluid Envelope generally works to decrease $\epsilon_{\rm max}$.
- Maximal triaxialities of Solar System Bodies close to observed values.