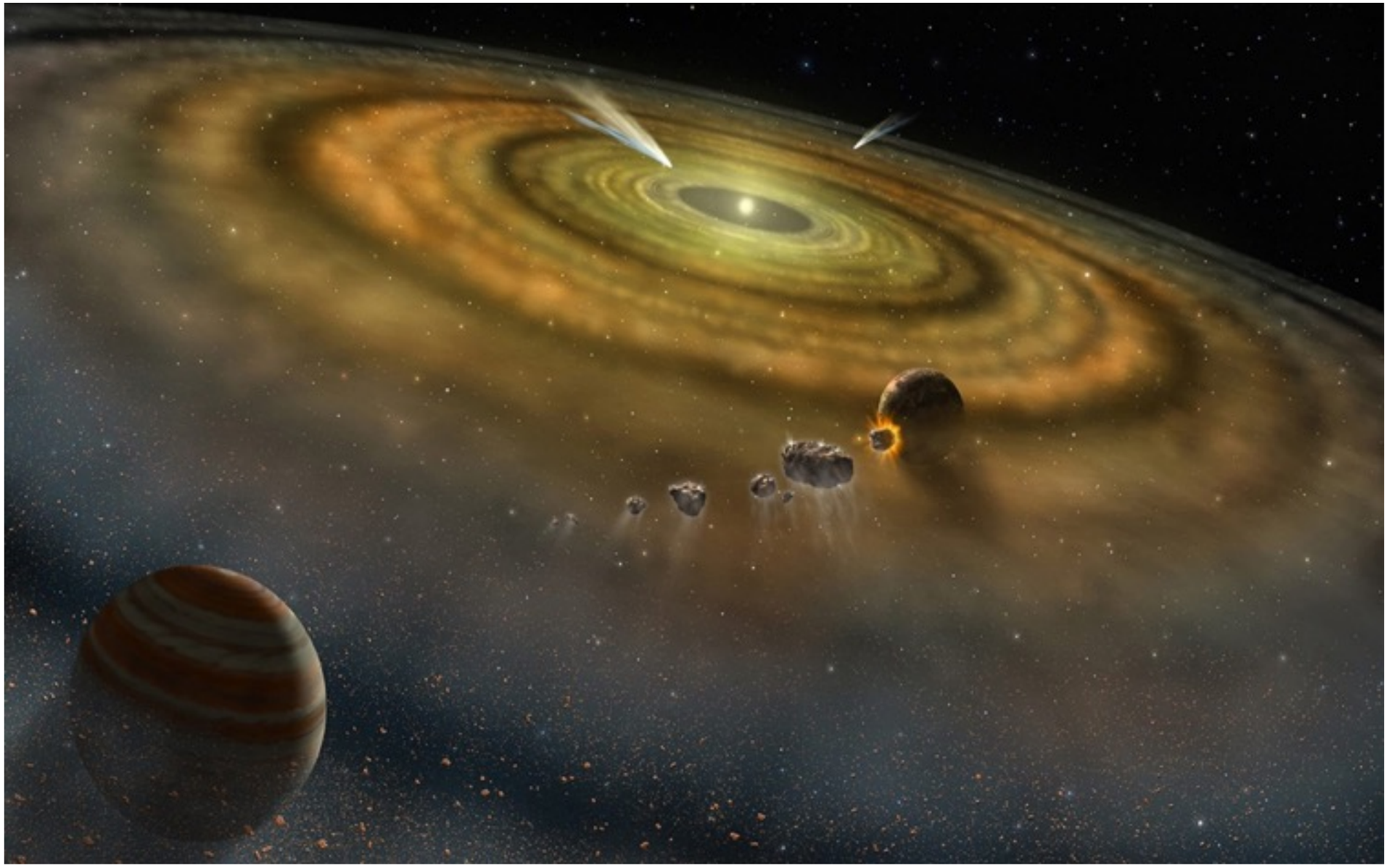

A Compressible, Vorticity-Preserving Hydrodynamical Scheme for Modeling Accretion Disk Flows

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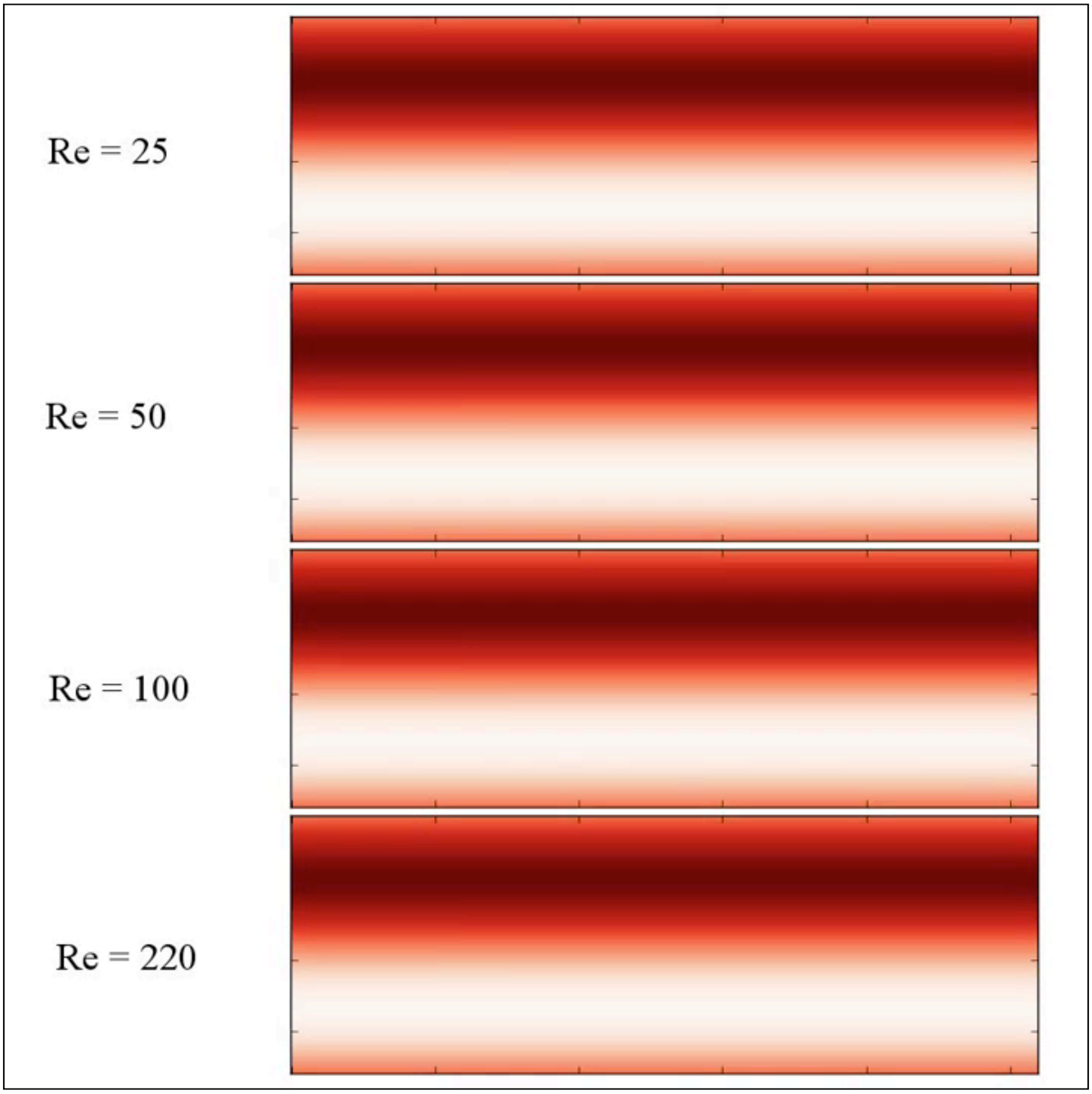
"Every rotating cosmic fluid that can be observed sufficiently closely displays either vortices or magnetic flux tubes on its surface; examples are tornadoes in the Earth's atmosphere, the Great Red Spot and other vortices in Jupiter's atmosphere, and sunspots. We suggest here that hot accretion disks also produce coherent objects"

-M. A. Abramowicz, *Vortices on Accretion Disks, Letters to Nature 1992*

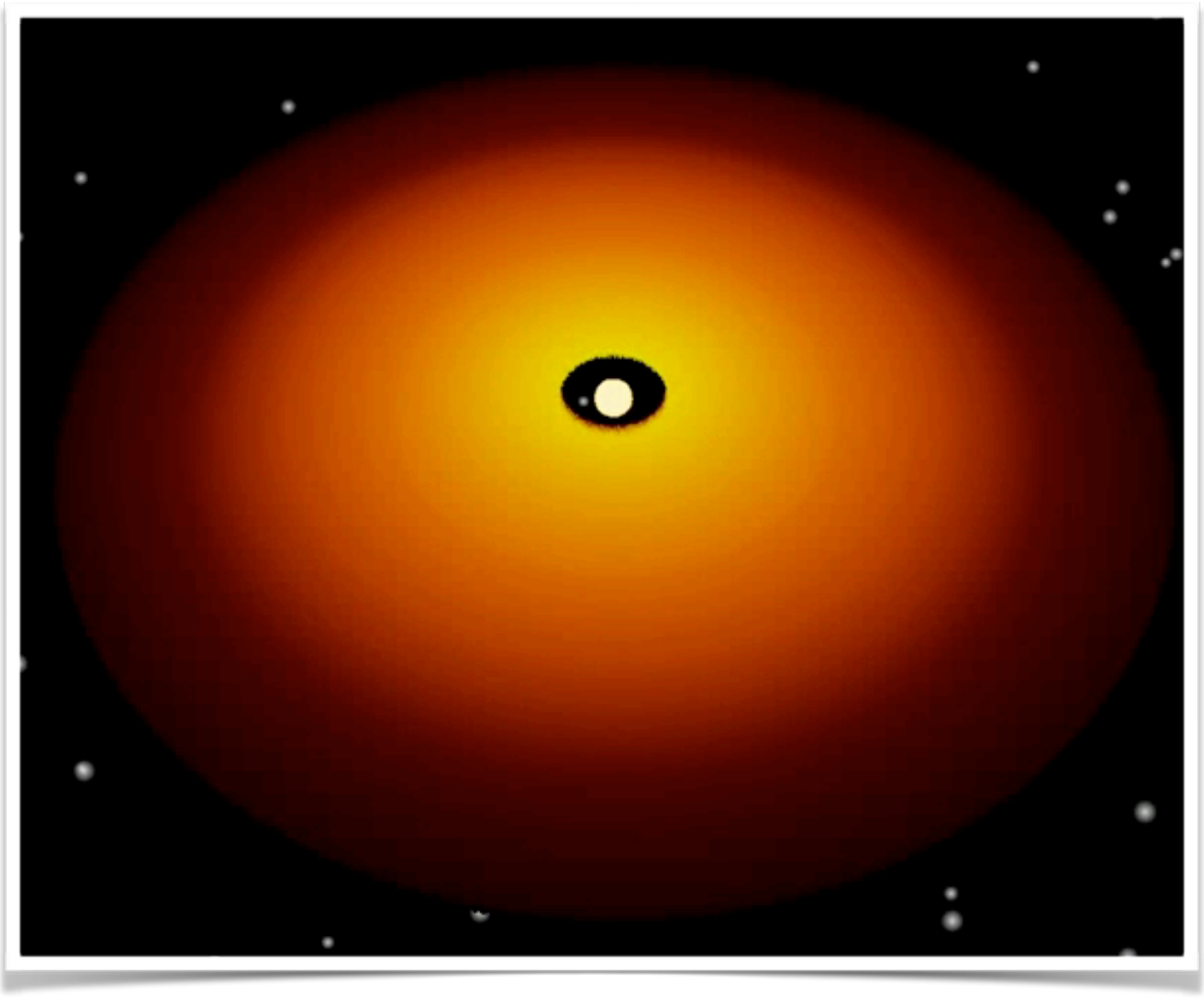
Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\vec{\nabla} p}{\rho} - \nabla \Phi + \nu \nabla^2 \mathbf{u}$$

$$Re = \frac{VL}{\nu}$$


Re = 25
Re = 50
Re = 100
Re = 220

Numerical Simulations



Evolution of Vorticity

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \,,$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\vec{\nabla} p}{\rho} - \nabla \Phi + \nu \nabla^2 \mathbf{u} \,,$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = -(\mathbf{u} \cdot \nabla)\boldsymbol{\omega} + (\boldsymbol{\omega} \cdot \vec{\nabla})\mathbf{V} - (\vec{\nabla} \cdot \mathbf{V})\boldsymbol{\omega} + \frac{\vec{\nabla}\rho \times \vec{\nabla} p}{\rho^2} \,.$$

Stable Fluids

$$\mathbf{w} = \mathbf{u} + \nabla q,$$

$$\nabla \cdot \mathbf{w} = \nabla^2 q.$$

$$\mathbf{u} = \mathbf{P}\mathbf{w} = \mathbf{w} - \nabla q.$$

Stam 2001

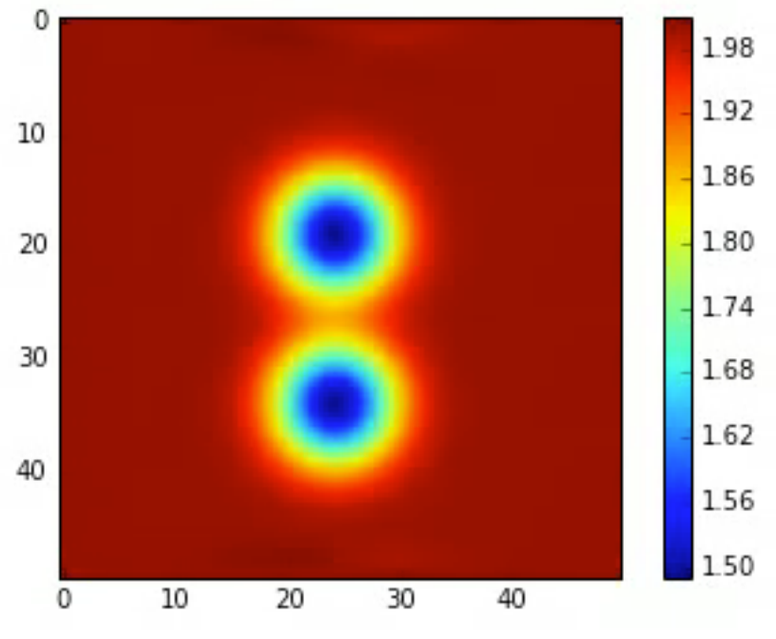
A Vorticity Preserving Compressible Scheme

$$w = \begin{bmatrix} p \\ u \\ v \end{bmatrix} \qquad f = f(w) = \begin{bmatrix} u \\ p \\ 0 \end{bmatrix} \qquad g = g(w) = \begin{bmatrix} v \\ 0 \\ p \end{bmatrix}$$

$$\partial_t w + \partial_x f + \partial_y g = \epsilon(\partial_{xx} w + \partial_{yy} w)$$

$$\partial_t \vec{V} + \nabla p = \epsilon \nabla^2(\vec{V})$$

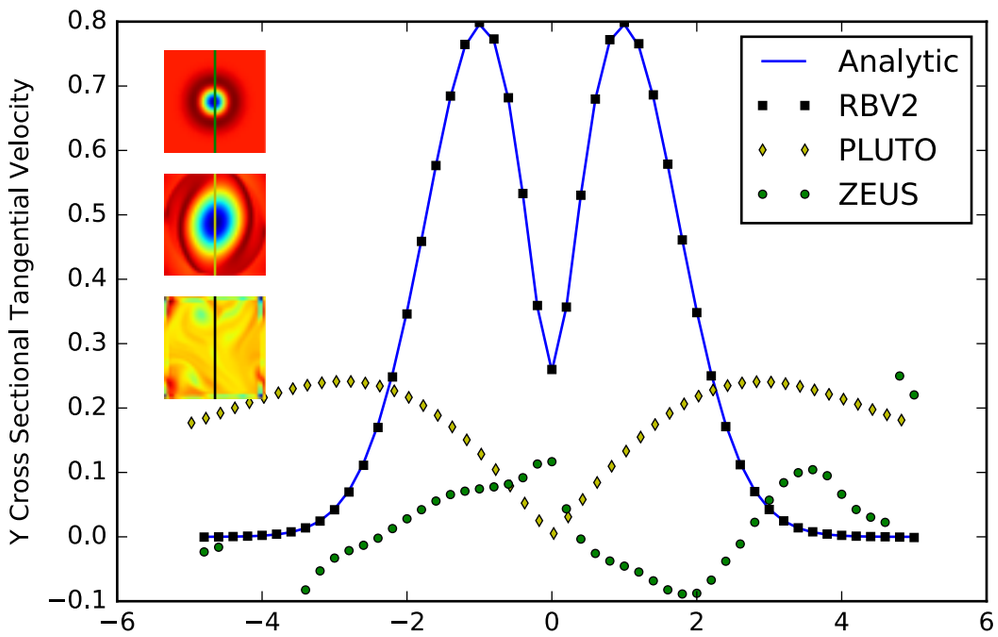
$$\partial_t \vec{\omega} = \epsilon \nabla^2(\omega)$$

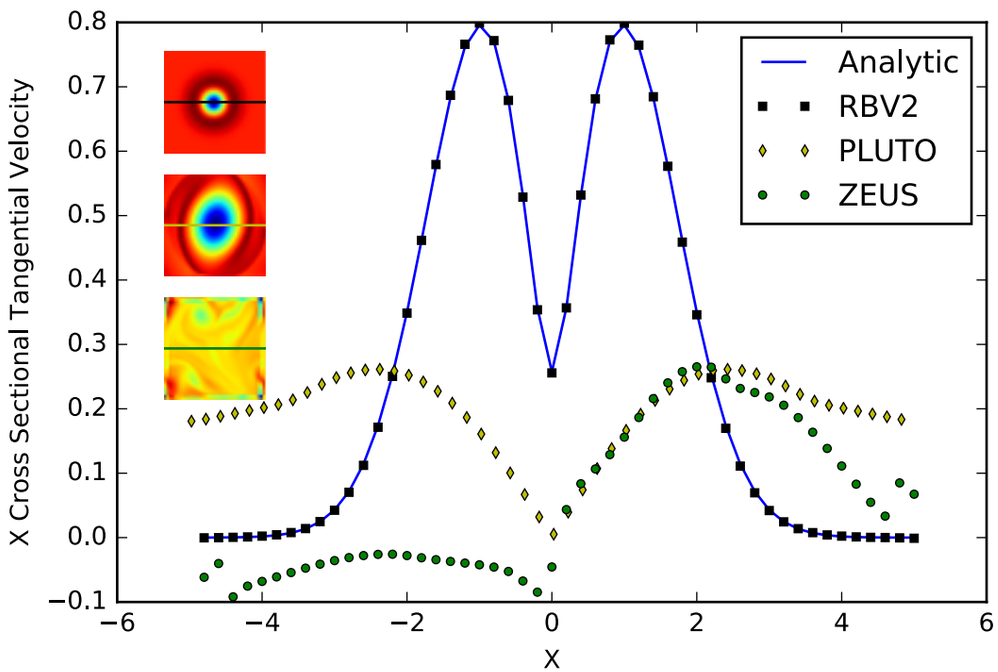


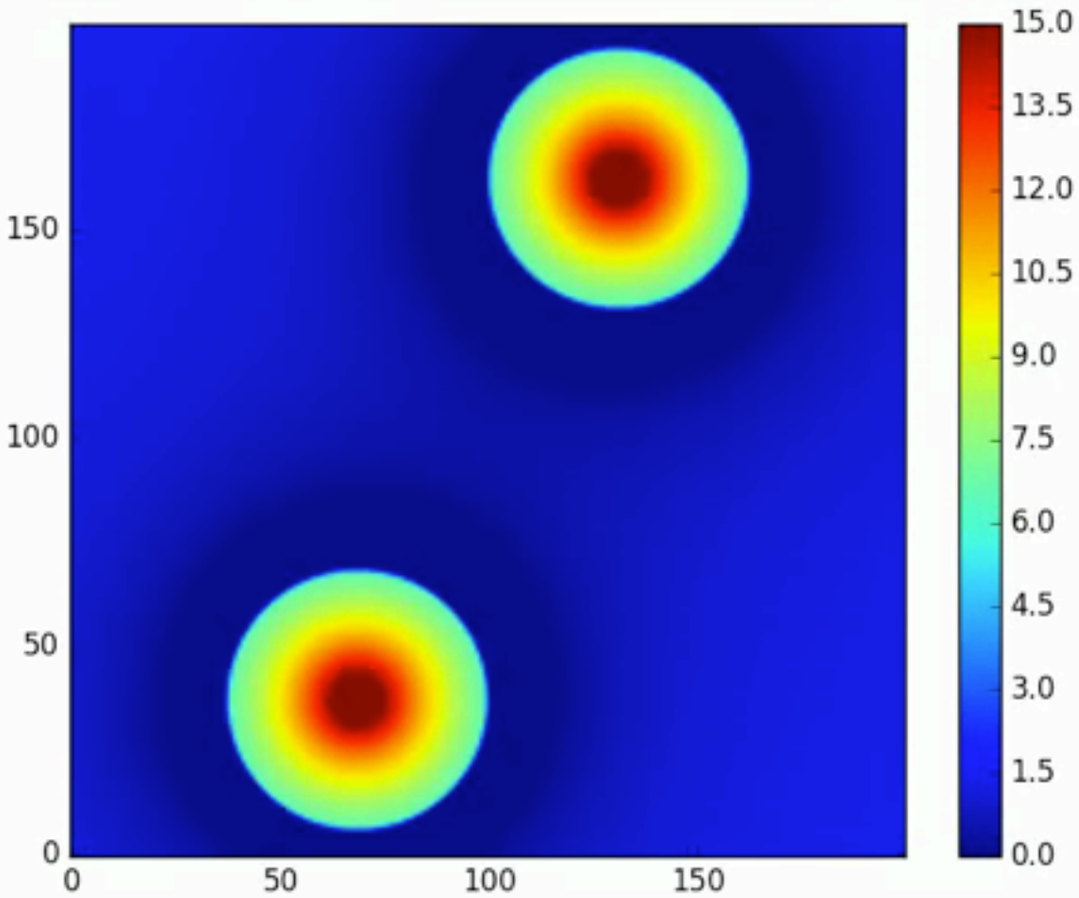
Lerat+ 2007

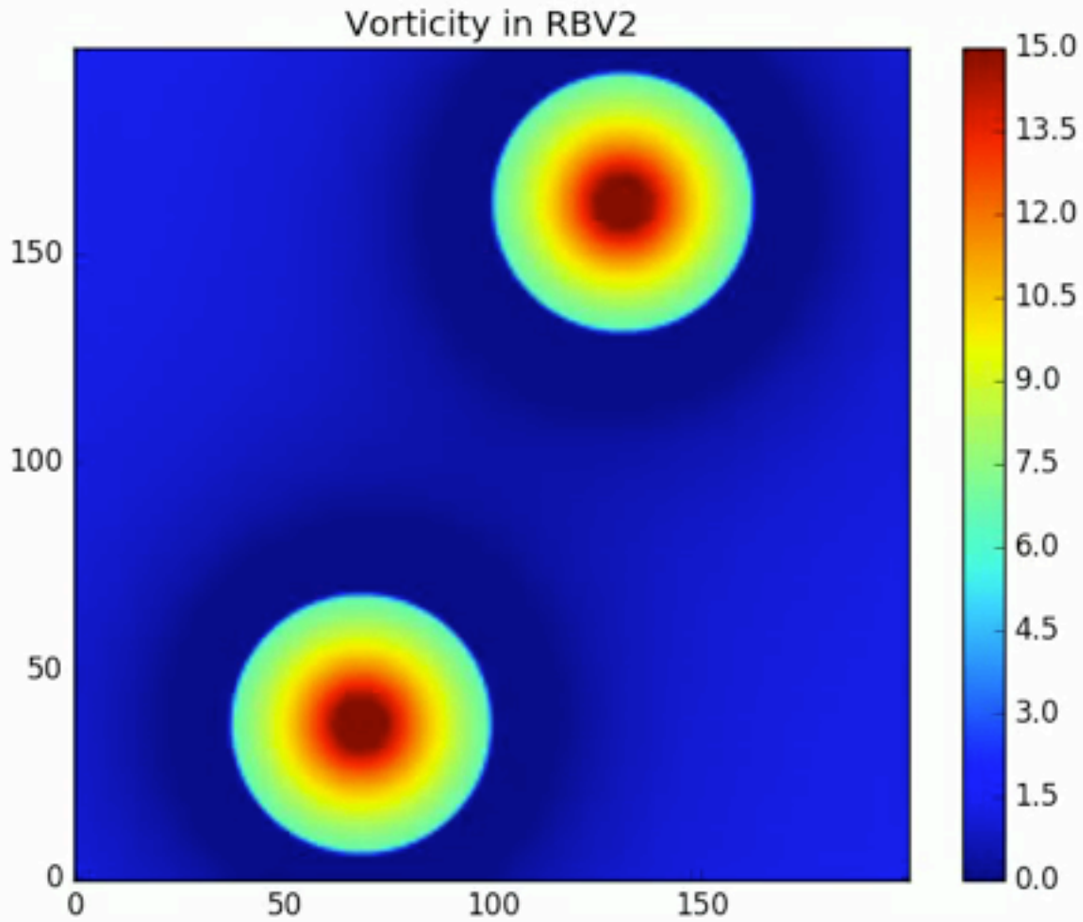
The Geostrophic Balance

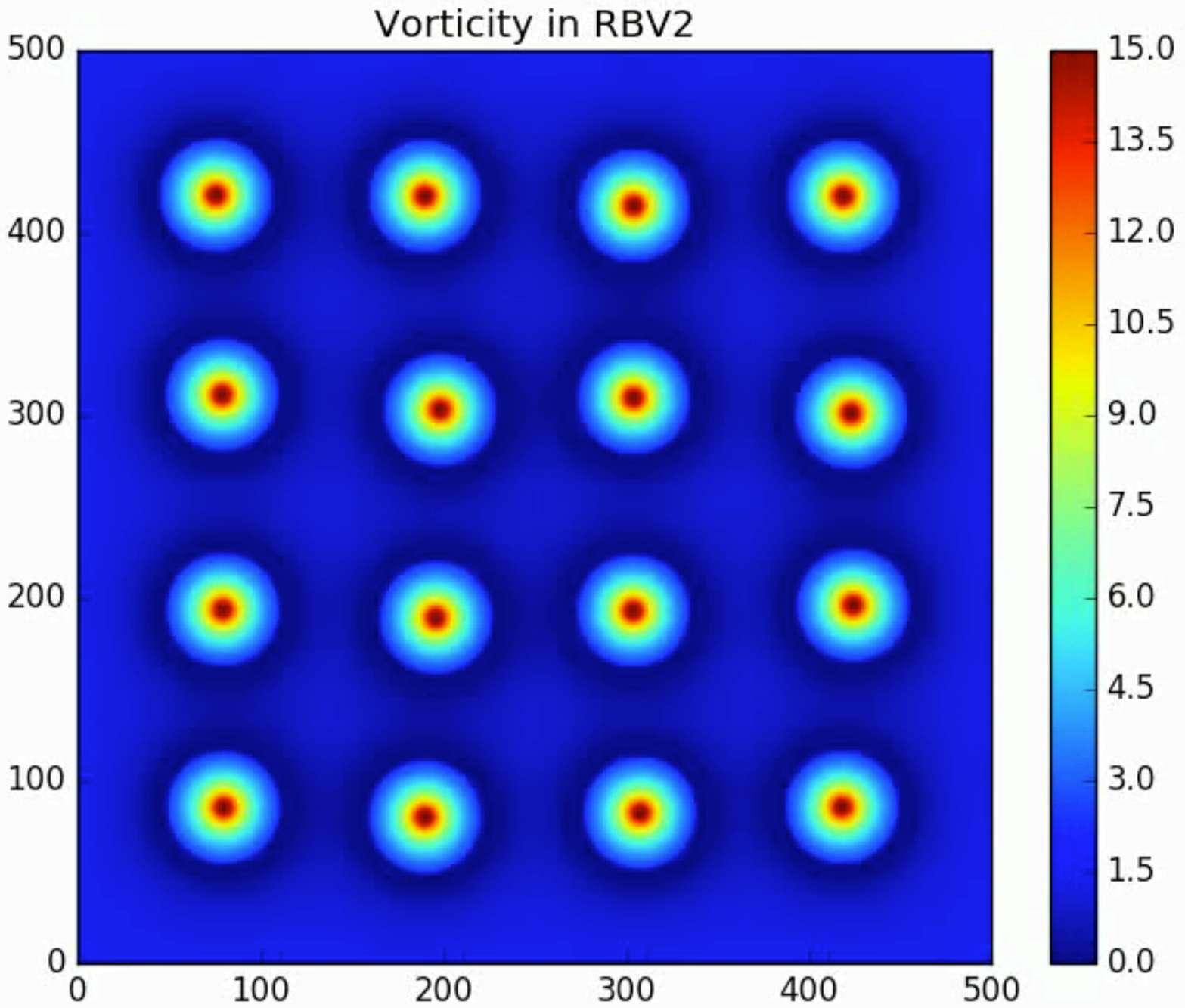
$$\left\| \frac{\nabla P}{\rho} \right\| = \frac{v^2}{r}$$






150
100
50
0
0
50
100
150
15.0
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0


Vorticity in RBV2
150
100
50
0
0
50
100
150
15.0
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0


Vorticity in RBV2
500
400
300
200
100
0
0
100
200
300
400
500
15.0
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0

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